NUMERICAL RELATIONSHIPS BETWEEN THE MASSES OF SOME ELEMENTARY PARTICLES.

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What we perceive are never unique properties of individual objects, but always and only properties that objects possess in common with other objects.

[...] the passage from concepts of "substance" to concepts of "function" is characteristic of the historical development of science. "Concepts-things" have gradually and often painstakingly given way to "relational concepts".

(Heinrich Klüver in the introduction to "The sensory order" by Friedrich A. von Hayek.)

Summary

The following expressions and formulas relate the masses of thirteen elementary particles to each other, with the aim of showing that their apparently random values are connected and interdependent.

The mass of each specific particle is a linear combination (in a generalized sense) of other particles, or the product of a numerical coefficient for a particle. In turn, the coefficients are combinations of mass ratios.

So, starting from the mass of each particle it is possible to find one or more "paths" that lead to each other mass.

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PART I – A relationship between the masses of proton, electron, μ lepton and charged π meson.

Let's consider the expression : $\frac{m\mu - e^{\pi} * me}{mp} + \frac{9}{10}$, which combines the masses of the μ lepton $(m\mu)$, of electron (me) and of proton (mp). For convenience we will call it $cosh(\theta_{m\pi^{\pm}})$, that is: hyperbolic cosine of $\theta_{m\pi^{\pm}}$. The reason for the name is that there is another expression: $\frac{me}{m\pi^{\pm}}$, which we will call $sinh(\theta_{m\pi^{\pm}})$, where $m\pi^{\pm}$ is the mass of the charged π meson, for which:

$$\left(\frac{m\mu - e^{\pi} * me}{mp} + \frac{9}{10}\right)^2 - \left(\frac{me}{m\pi^{\pm}}\right)^2 = 1 \qquad [1.1]$$

This equality is formally analogous to: $cosh^2(\theta_{m\pi^{\pm}}) - sinh^2(\theta_{m\pi^{\pm}}) = 1$ of the hyperbolic cosine and sine, and therefore we will exploit this correspondence extensively to simplify various formulas.

Expression [1.1] therefore highlights a numerical relationship between the values of the electron, proton, μ lepton and charged π meson masses.

[The values of the masses are taken from the tables published by NIST, Codata and / or Particle Data Group (PDG). All the results obtained fall within the limits of the experimental errors with which the masses are known.]

It is also evident, based on the definition given, that:

$$sinh(\theta_{m\pi^{\pm}}) = \sqrt{\left(\frac{m\mu - e^{\pi} * me}{mp} + \frac{9}{10}\right)^2 - 1} \qquad [1.2]$$

and that it is also: $$cosh(\theta_{m\pi^{\pm}}) = \sqrt{1 + \left(\frac{me}{m\pi^{\pm}}\right)^2} \qquad [1.3]$$

[We will often use from here on one form or the other of the hyperbolic or trigonometric sines and cosines depending on whether we are interested in making the masses of certain particles appear explicitly in the formulas or at least excluding them from direct references, given their intertwined relationships.]

We can therefore derive from [1.1], for example, the mass of the proton as a function of the other three particles:

$$mp = \frac{m\mu - e^{\pi} * me}{\sqrt{1 + \left(\frac{me}{m\pi^{\pm}}\right)^2} - \frac{9}{10}} \qquad [1.4]$$

and in a more concise form: $$mp = \frac{m\mu - e^{\pi} * me}{cosh(\theta_{m\pi^{\pm}}) - \frac{9}{10}} \qquad [1.5]$$

[The multiplication sign (*) was later maintained or omitted depending on the need for clarity or convenience.]

Since $cosh(\theta_{m\pi^{\pm}})$ is a dimensionless number, we can highlight how here mp results in a linear combination of $m\mu$ and $me,$ in which $m\pi^{\pm}$ contributes to determine the value of the coefficients:

$$mp = \left(\frac{1}{cosh(\theta_{m\pi^{\pm}}) - \frac{9}{10}}\right) * m\mu - \left(\frac{e^{\pi}}{cosh(\theta_{m\pi^{\pm}}) - \frac{9}{10}}\right) * me \qquad [1.6]$$

The constant $\frac{9}{10}$ can be eliminated, as we will see later, provided that other particles and more complicated expressions are introduced into the formulas.

Similarly, due to [1.1] , the mass of the μ lepton is:

$$m\mu = mp\left(\sqrt{1 + \left(\frac{me}{m\pi^{\pm}}\right)^2} - \frac{9}{10}\right) + e^{\pi} * me \qquad [1.7]$$

or: $$m\mu = mp\left(cosh(\theta_{m\pi^{\pm}}) - \frac{9}{10}\right) + e^{\pi} * me \qquad [1.8]$$

while that of $\pi^{\pm}$ meson is: $$m\pi^{\pm} = \frac{me}{\sqrt{\left(\frac{m\mu - e^{\pi} * me}{mp} + \frac{9}{10}\right)^2 - 1}} \qquad [1.9]$$

and therefore, simply: $$m\pi^{\pm} = \frac{me}{sinh(\theta_{m\pi^{\pm}})} \qquad [1.10]$$

The concise form will be more advantageous later, and will allow to better compare the structure of some formulas.

As for the electron, the expression of its mass - seen as unknown and as a function of the other three masses considered here -, gives rise to a second degree equation:

$$\left(e^{2\pi} - \frac{mp^2}{m\pi^{\pm 2}}\right)me^2 - \left(\left(2m\mu + \frac{9}{5}mp\right)e^{\pi}\right)me + \left(\frac{9}{10}mp + m\mu\right)^2 - mp^2 = 0$$

which implies: $me_{1,2} =$

$$= \frac{\left(2m\mu + \frac{9}{5}mp\right)e^{\pi} \pm \sqrt{\left(\left(2m\mu + \frac{9}{5}mp\right)e^{\pi}\right)^2 - 4 * \left(e^{2\pi} - \frac{mp^2}{m\pi^{\pm 2}}\right) * \left(\left(\frac{9}{10}mp + m\mu\right)^2 - mp^2\right)}}{2\left(e^{2\pi} - \frac{mp^2}{m\pi^{\pm 2}}\right)}$$

which has two positive solutions: the second is precisely the known mass of the electron, the other a much larger mass, which is worth $1{,}5896586(18)*10^{-28}$ kg. It has no matching between the known masses.

However, we will soon see a simpler formulation of the electron mass.

PART II – The neutron, the neutral π e K mesons and additional formulas for $m\pi^{\pm}$ and me.

Now consider the expression: $$\frac{m\mu + e^{\pi} * me}{2mn} + \frac{9}{10} \qquad [2.1]$$

in which mn is the neutron mass. We will call it $cos(\varphi)$**.** The reason, similarly to what we saw above, lies in the fact that there is another expression, which we will call $sin(\varphi)$**:**

$$sin(\varphi) = \frac{m\pi^0}{mK^0}\sqrt{5}\,\frac{(5m\pi^0 - 4m\pi^{\pm})}{(4m\pi^0 - 2m\pi^{\pm})} \qquad [2.2]$$

where $m\pi^0$ is the mass of the neutral π meson and mK^0 that of the neutral K meson, for which the trigonometric relationship exists: $\cos(\varphi)^2 + \sin(\varphi)^2 = 1$, that is:

$$\left(\frac{m\mu + e^{\pi} * me}{2mn} + \frac{9}{10}\right)^2 + \left(\frac{m\pi^0}{mK^0}\sqrt{5}\ \frac{(5m\pi^0 - 4m\pi^{\pm})}{(4m\pi^0 - 2m\pi^{\pm})}\right)^2 = 1 \qquad [2.3]$$

From this relationship we derive the equation for the mass of the neutron:

$$mn = \frac{m\mu + e^{\pi} * me}{2\left(\sqrt{1 - \left(\frac{m\pi^0}{mK^0}\sqrt{5}\ \frac{(5m\pi^0 - 4m\pi^{\pm})}{(4m\pi^0 - 2m\pi^{\pm})}\right)^2} - \frac{9}{10}\right)} \qquad [2.4]$$

Since $cos(\varphi)$ [2.1] can be written as: $\sqrt{1 - sin(\varphi)^2}$, i.e., it is also:

$$cos(\varphi) = \sqrt{1 - \left(\frac{m\pi^0}{mK^0}\sqrt{5}\ \frac{(5m\pi^0 - 4m\pi^{\pm})}{(4m\pi^0 - 2m\pi^{\pm})}\right)^2} \qquad [2.5]$$

we rewrite the neutron mass as: $mn = \frac{m\mu + e^{\pi} * me}{2\left(\cos(\varphi) - \frac{9}{10}\right)}$ [2.6]

which highlights the similarity with the formula of the proton mass mp in [1.5].

Using the relation [1.8] and replacing $e^{\pi} * me$ in [2.6] with: $m\mu - mp\left(cosh(\theta_{\mathrm{m}\pi^{\pm}}) - \frac{9}{10}\right)$, we get:

$$mn = \frac{2m\mu - mp\left(cosh(\theta_{\mathrm{m}\pi^{\pm}}) - \frac{9}{10}\right)}{2\left(\cos(\varphi) - \frac{9}{10}\right)} \qquad [2.7]$$

or:

$$mn = \frac{mp\left(cosh(\theta_{\mathrm{m}\pi^{\pm}}) - \frac{9}{10}\right) + 2e^{\pi} * me}{2\left(\cos(\varphi) - \frac{9}{10}\right)} \qquad [2.8]$$

where $m\mu$ was replaced by: $mp\left(cosh(\theta_{m\pi^{\pm}}) - \frac{9}{10}\right) + e^{\pi} * me$.

From the synthesis of the two formulas [2.7] and [2.8], mp can be expressed as:

$$mp = \frac{2mn * (m\mu - e^{\pi} * me)}{2mn * \left(cosh(\theta_{\mathrm{m\pi}^{\pm}}) - \cos(\varphi)\right) + (m\mu + e^{\pi} * me)} \qquad [2.9]$$

and inversely:

$$mn = \frac{mp * (m\mu + e^{\pi} * me)}{2\left(mp * \left(\cos(\varphi) - cosh(\theta_{\mathrm{m\pi}^{\pm}})\right) + (m\mu - e^{\pi} * me)\right)} \qquad [2.10]$$

Note, in particular, the various changes of sign in these two last formulas and that, as promised, the constant $\frac{9}{10}$ has been eliminated. Indeed, it is also:

$$\frac{9}{10} = \frac{2mn * cos(\varphi) + mp * cosh(\theta_{m\pi^{\pm}}) - 2m\mu}{2mn + mp}$$

as well as, of course:

$$\frac{9}{10} = cos(\varphi) - \frac{m\mu + e^{\pi} * me}{2mn} \quad \text{and:} \quad \frac{9}{10} = cos(\theta_{m\pi^{\pm}}) - \frac{m\mu - e^{\pi} * me}{mp}$$

In turn, the electron and lepton μ masses can be written, using [2.6] (and [2.5] for $cos(\varphi)$), as:

$$me = \frac{2mn * \left(cos(\varphi) - \frac{9}{10}\right) - m\mu}{e^{\pi}} \qquad [2.11]$$

$$m\mu = 2mn * \left(cos(\varphi) - \frac{9}{10}\right) - e^{\pi} * me \qquad [2.12]$$

and also (from [2.6] and from the already used replacement of $e^{\pi} * me$) as:

$$m\mu = \frac{1}{2}\left(cosh(\theta_{m\pi^{\pm}}) - \frac{9}{10}\right) * mp + \left(cos(\varphi) - \frac{9}{10}\right) * mn \qquad [2.13]$$

which highlights $m\mu$ as a linear combination of mp and mn.

From [2.3] we can make mK^0 explicit as a function of $m\pi^0$, $m\pi^{\pm}$, $m\mu$, me and mn :

$$mK^0 = \frac{m\pi^0 * \sqrt{5}\ \frac{(5m\pi^0 - 4m\pi^{\pm})}{(4m\pi^0 - 2m\pi^{\pm})}}{\sqrt{1 - \left(\frac{m\mu + e^{\pi} * me}{2mn} + \frac{9}{10}\right)^2}} \qquad [2.14]$$

Written $sin(\varphi)$ as $\sqrt{1 - cos(\varphi)^2}$, that is (for [2.1], [2.2] and [2.3]):

$$sin(\varphi) = \sqrt{1 - \left(\frac{m\mu + e^{\pi} * me}{2mn} + \frac{9}{10}\right)^2} \qquad [2.15]$$

mK^0 becomes:

$$mK^0 = \frac{m\pi^0 * \sqrt{5}\ \frac{(5m\pi^0 - 4m\pi^{\pm})}{(4m\pi^0 - 2m\pi^{\pm})}}{sin(\varphi)} \qquad [2.16]$$

If we now make explicit [2.16] for $m\pi^0$, we get a second degree equation:

$$5\sqrt{5}\, m{\pi^0}^2 - \left(4mK^0 * sin(\varphi) + 4\sqrt{5}\, m\pi^{\pm}\right)m\pi^0 + 2mK^0 * sin(\varphi) * m\pi^{\pm} = 0$$

Dividing it by $\sqrt{5}$, we have:

$$5\, m{\pi^0}^2 - \left(\frac{4}{\sqrt{5}}mK^0 * sin(\varphi) + 4\, m\pi^{\pm}\right)m\pi^0 + \frac{2}{\sqrt{5}}mK^0 * sin(\varphi) * m\pi^{\pm} = 0$$

In this way we have the value 10 as the denominator of the solution formula (we will see why):

$$m{\pi^0}_{1,2} = \frac{\left(\frac{4}{\sqrt{5}}mK^0 * sin(\varphi) + 4\, m\pi^{\pm}\right) \pm \sqrt{\left(\frac{4}{\sqrt{5}}mK^0 * sin(\varphi) + 4\, m\pi^{\pm}\right)^2 - 4*5*\frac{2}{\sqrt{5}}mK^0 * sin(\varphi) * m\pi^{\pm}}}{10}$$

Of the two solutions, both positive, ${m\pi^0}_1$ coincides with the known value of $m\pi^0$, while ${m\pi^0}_2$ = 4.4501800(50)*10^(-29). The latter value has no matching among the known masses, although it shows some "relationships".

Now, since ${m\pi^0}_1 \equiv m\pi^0$, we can write:

$$\frac{1}{10} = \frac{m\pi^0}{\left(\frac{4}{\sqrt{5}} mK^0 * sin(\varphi) + 4\ m\pi^{\pm}\right) + \sqrt{\left(\frac{4}{\sqrt{5}} mK^0 * sin(\varphi) + 4\ m\pi^{\pm}\right)^2 - 4*5*\frac{2}{\sqrt{5}} mK^0 * sin(\varphi) * m\pi^{\pm}}}$$

Obviously, $\frac{1}{10} = 1 - \frac{9}{10}$, and here is the $\frac{9}{10}$ reappear: if we went to replace it in some of the expressions in which it is present (for example in [1.4]), inserting it in the form:

$$1 - \frac{m\pi^0}{\left(\frac{4}{\sqrt{5}} mK^0 * sin(\varphi) + 4\ m\pi^{\pm}\right) + \sqrt{\left(\frac{4}{\sqrt{5}} mK^0 * sin(\varphi) + 4\ m\pi^{\pm}\right)^2 - 4*5*\frac{2}{\sqrt{5}} mK^0 * sin(\varphi) * m\pi^{\pm}}} \qquad [2.17]$$

in addition to the remarkable complication of the formulas, we would notice the swirling "nesting" of particles within other particles.

From [2.14] we are now able to express a second version of $m\pi^{\pm}$:

$$m\pi^{\pm} = \frac{5\sqrt{5} * {m\pi^0}^2 - \sqrt{1 - \left(\frac{m\mu + e^{\pi} * me}{2mn} + \frac{9}{10}\right)^2} * mK^0 * 4m\pi^0}{2 * \left(2\sqrt{5} * m\pi^0 - \sqrt{1 - \left(\frac{m\mu + e^{\pi} * me}{2mn} + \frac{9}{10}\right)^2} * mK^0\right)} \qquad [2.18]$$

or in the equivalent but more concise form:

$$m\pi^{\pm} = \frac{5\sqrt{5} * {m\pi^0}^2 - 4sin(\varphi) * mK^0 * m\pi^0}{4\sqrt{5} m\pi^0 - 2sin(\varphi) * mK^0} \qquad [2.19]$$

which, written as follows:

$$m\pi^{\pm} = \left(\frac{5\sqrt{5}}{4\sqrt{5} - 2sin(\varphi) * \frac{mK^0}{m\pi^0}}\right) * m\pi^0 - \left(\frac{4sin(\varphi)}{4\sqrt{5} - 2sin(\varphi) * \frac{mK^0}{m\pi^0}}\right) * mK^0 \qquad [2.20]$$

more explicitly shows $m\pi^{\pm}$ as a linear combination of $m\pi^0$ and mK^0 .

Also, remembering that $m\pi^{\pm} = \frac{me}{sinh(\theta_{m\pi^{\pm}})}$ (ref. [1.10]), if from [2.7] we take:

$$cosh(\theta_{m\pi^{\pm}}) = \frac{2m\mu - 2mn * \left(cos(\varphi) - \frac{9}{10}\right)}{mp} + \frac{9}{10} \qquad [2.21]$$

we can write $sinh(\theta_{m\pi^{\pm}})$ in a version independent from $me,$ that is:

$$sinh(\theta_{m\pi^{\pm}}) = \sqrt{\left(\frac{2m\mu - 2mn * \left(cos(\varphi) - \frac{9}{10}\right)}{mp} + \frac{9}{10}\right)^2 - 1} \qquad [2.22]$$

(where $cos(\varphi)$ appears as in [2.5]) . If we multiply [2.22] by [2.20] , we get a further formula for me, that is:

$$me = \left(\frac{5\sqrt{5} * sinh(\theta_{m\pi^{\pm}})}{4\sqrt{5} - 2sin(\varphi) * \frac{mK^0}{m\pi^0}}\right) * m\pi^0 - \left(\frac{4sin(\varphi) * sinh(\theta_{m\pi^{\pm}})}{4\sqrt{5} - 2sin(\varphi) * \frac{mK^0}{m\pi^0}}\right) * mK^0 \qquad [2.23]$$

with $sin(\varphi)$ brought back to the original version [2.2] for not explicitly having me on the right side of the formula, also evidently a linear combination of $m\pi^0$ and mK^0.

PART III – A further formula for $m\pi^0$, introducing the mass of $K^{\pm}$ meson.

Let's take a step back now. We have seen that $m\pi^{\pm}$ in the first version ([1.9]) is:

$$m\pi^{\pm} = \frac{me}{\sqrt{\left(\frac{m\mu - e^{\pi} * me}{mp} + \frac{9}{10}\right)^2 - 1}}$$

At this point we can ask whether $m\pi^0$ can also be represented using a similar expression. After all, the masses of the two mesons do not differ much, so it is reasonable to think that some variation in the formula of the $m\pi^{\pm}$ leads to that of the $m\pi^0$.

The answer is yes, and the variation consists in changing the exponent of e^{π}.

This modification has a cost, i.e. the introduction of a further mass, that of the charged meson $K^{\pm}$. In fact, it is necessary to replace the exponent π with:

$$\pi - \left(\frac{mp}{mK^{\pm}}\right) * \frac{cosh(\theta_{\mathrm{m\pi}^{\pm}}) - \frac{9}{10}}{cosh(\theta_{\mathrm{m\pi}^{\pm}}) + \frac{9}{10}} + \frac{1}{10}$$

With this value, which we will abbreviate in π-Φ, the new formulation of the $m\pi^0$ becomes:

$$m\pi^0 = \frac{me}{\sqrt{\left(\frac{m\mu - e^{(\pi-\Phi)} * me}{mp} + \frac{9}{10}\right)^2 - 1}}$$

formula whose structure is similar to that of $m\pi^{\pm}$, and which in full becomes:

$$m\pi^0 = \frac{me}{\sqrt{\left(\frac{m\mu - e^{\left(\pi - \left(\frac{mp}{mK^{\pm}}\right) * \frac{cosh(\theta_{m\pi^{\pm}}) - \frac{9}{10}}{cosh(\theta_{m\pi^{\pm}}) + \frac{9}{10}} + \frac{1}{10}\right)} * me}{mp} + \frac{9}{10}\right)^2 - 1}} \qquad [3.1]$$

Based on previously established relationships (see [1.5]),

$$mp * \frac{cosh(\theta_{m\pi^{\pm}}) - \frac{9}{10}}{cosh(\theta_{m\pi^{\pm}}) + \frac{9}{10}} \quad \text{is equivalent to:} \quad \frac{m\mu - e^{\pi} * me}{cosh(\theta_{\mathrm{m\pi}^{\pm}}) + \frac{9}{10}} \quad ,$$

so that a further formulation for $m\pi^0$ is:

$$m\pi^0 = \frac{me}{\sqrt{\left(\frac{m\mu - e^{\left(\pi - \frac{m\mu - e^{\pi} * me}{mK^{\pm} * \left(cosh(\theta_{m\pi^{\pm}}) + \frac{9}{10}\right)} + \frac{1}{10}\right)} * me}{mp} + \frac{9}{10}\right)^2 - 1}} \qquad [3.2]$$

which once again highlights the nesting of the masses.

Also in this formula, by analogy with $m\pi^{\pm}$, we define the whole denominator equal to $sinh(\theta_{m\pi^0})$ and write, in much shorter form:

$$m\pi^0 = \frac{me}{sinh(\theta_{m\pi^0})} \qquad [3.3]$$

Consequently,

$$cosh(\theta_{m\pi^0}) = \frac{m\mu - e^{\left(\pi - \frac{m\mu - e^{\pi} * me}{mK^{\pm} * \left(\cosh(\theta m\pi^{\pm}) + \frac{9}{10}\right)} + \frac{1}{10}\right)} * me}{mp} + \frac{9}{10} \qquad [3.4]$$

and therefore: $cosh^2(\theta_{m\pi^0}) - sinh^2(\theta_{m\pi^0}) = 1$, that is:

$$\left(\frac{m\mu - e^{\left(\pi - \frac{m\mu - e^{\pi} * me}{mK^{\pm} * \left(\cosh(\theta m\pi^{\pm}) + \frac{9}{10}\right)} + \frac{1}{10}\right)} * me}{mp} + \frac{9}{10}\right)^2 - \left(\frac{me}{m\pi^0}\right)^2 = 1 \qquad [3.5]$$

From [3.4] we obtain, for instance, further formulas for mp e $m\mu$, where however:

$$cosh(\theta_{m\pi^0}) = \sqrt{1 + sinh(\theta_{m\pi^0})^2} = \sqrt{1 + \left(\frac{me}{m\pi^0}\right)^2} :$$

$$mp = \frac{m\mu - e^{\left(\pi - \frac{m\mu - e^{\pi} * me}{mK^{\pm} * \left(\cosh(\theta m\pi^{\pm}) + \frac{9}{10}\right)} + \frac{1}{10}\right)} * me}{cosh(\theta_{m\pi^0}) - \frac{9}{10}} \qquad [3.6]$$

$$m\mu = mp\left(cosh(\theta_{m\pi^0}) - \frac{9}{10}\right) + e^{\left(\pi - \frac{m\mu - e^{\pi} * me}{mK^{\pm} * \left(\cosh(\theta m\pi^{\pm}) + \frac{9}{10}\right)} + \frac{1}{10}\right)} * me \qquad [3.7]$$

Explaining now the [3.1] for $mK^{\pm}$, we reach:

$$mK^{\pm} = \frac{mp * \frac{cosh(\theta_{m\pi^{\pm}}) - \frac{9}{10}}{cosh(\theta_{m\pi^{\pm}}) + \frac{9}{10}}}{\pi + \frac{1}{10} - ln\left(\frac{m\mu - mp * \left(\sqrt{1 + \left(\frac{me}{m\pi^{0}}\right)^{2}} - \frac{9}{10}\right)}{me}\right)} \qquad [3.8]$$

that we can rewrite as:

$$mK^{\pm} = \frac{mp * \frac{cosh(\theta_{m\pi^{\pm}}) - \frac{9}{10}}{cosh(\theta_{m\pi^{\pm}}) + \frac{9}{10}}}{\pi + \frac{1}{10} - ln\left(\frac{m\mu - mp * \left(cosh(\theta_{m\pi^{0}}) - \frac{9}{10}\right)}{me}\right)} \qquad [3.9]$$

or as:

$$mK^{\pm} = \frac{\frac{m\mu - e^{\pi} * me}{cosh(\theta_{m\pi^{\pm}}) + \frac{9}{10}}}{\pi + \frac{1}{10} - ln\left(\frac{m\mu - mp * \left(cosh(\theta_{m\pi^{0}}) - \frac{9}{10}\right)}{me}\right)} \qquad [3.10]$$

A further implicit equation for $mK^{\pm}$ is obtained by exploiting some of the relations obtained so far:

$$\frac{m\mu - mp * (cosh(\theta_{m\pi^{0}}) - \frac{9}{10})}{m\mu - mp * (cosh(\theta_{m\pi^{\pm}}) - \frac{9}{10})} = e^{\left(\frac{1}{10} - \frac{m\mu - e^{\pi} * me}{mK^{\pm} * \left(cosh(\theta_{m\pi^{\pm}}) + \frac{9}{10}\right)}\right)} \qquad [3.11]$$

PART IV – The masses of particles η and τ.

The particle we will now consider is η, whose mass we will denote by $m\eta$. This is its formula:

$$m\eta = 2me * e^{\left(2\pi + \left(\frac{mp^2+me^2}{mp^2-me^2}\right) * \frac{me^2}{mp^2} * \frac{c}{9\pi^2} - 1\right)} \qquad [4.1]$$

(The constant c is the value of the speed of light.)

We now come to the expression of the mass of the largest of the leptons, the particle τ ($m\tau$), which has a structure similar to [4.1]:

$$m\tau = 6me * e^{\left(2\pi + \frac{e^{(2\pi+B-1)} * me - m\mu}{mp} - cosh(\theta_{m\pi^{\pm}}) + \frac{9}{10}\right)} \qquad [4.2]$$

where factor B on the exponent holds: $B = \frac{\sqrt{1+\left(\frac{me}{m\pi^0}\right)^2}}{\sqrt{1+\left(\frac{me}{mK^0}\right)^2}}$

Since: $cosh(\theta_{m\pi^{\pm}}) = \frac{m\mu - e^{\pi} * me}{mp} + \frac{9}{10}$

we can also rewrite $m\tau$ as:

$$m\tau = 6me * e^{\left(2\pi + \frac{(e^{(2\pi+B-1)} + e^{\pi}) * me - 2m\mu}{mp}\right)} \qquad [4.3]$$

So, in both particles η and τ the dimension of mass is provided by the electron.

PART V – Three members of the Λ family $(\Lambda_s^0\ ,\ \Lambda_c^+\ ,\ \Lambda_b^0)$

Let's take the exponential in the formula of $m\eta$, that is:

$$e^{\left(2\pi+\left(1+\frac{me^2}{mp^2}\right)*\left(\frac{me^2}{mp^2-me^2}\right)*\frac{c}{9\pi^2}-1\right)} \qquad [5.1]$$

and replace $1+\frac{me^2}{mp^2}$ with $1+\frac{me^2}{m\pi^{\pm 2}}$ (that is, with $cosh^2(\theta_{m\pi^\pm})$).

We get:

$$e^{\left(2\pi+\left(1+\frac{me^2}{m\pi^{\pm 2}}\right)*\left(\frac{me^2}{mp^2-me^2}\right)*\frac{c}{9\pi^2}-1\right)} \qquad [5.2]$$

If we now divide the modified exponential [5.2] with the original one [5.1] and square the result, we obtain:

$$e^{2*\left(\frac{me^2}{m\pi^{\pm 2}}-\frac{me^2}{mp^2}\right)*\left(\frac{me^2}{mp^2-me^2}\right)*\frac{c}{9\pi^2}} \qquad [5.3]$$

Now, there is a numerical equivalence whereby:

$$\frac{m\mu}{2m\Lambda_c^+ + m\Lambda_s^0 - m\Lambda_b^0} * \left(\frac{mK^\pm - m\mu}{mK^\pm + m\mu}\right) = e^{2*\left(\frac{me^2}{m\pi^{\pm 2}}-\frac{me^2}{mp^2}\right)*\left(\frac{me^2}{mp^2-me^2}\right)*\frac{c}{9\pi^2}} \qquad [5.4]$$

where $m\Lambda_s^0$, $m\Lambda_c^+$, $m\Lambda_b^0$ are the masses of Λ_s^0 , Λ_c^+ , Λ_b^0 respectively.

From [5.4] one can therefore obtain the mass of each of the Λ particles as a function of the others.

If we put: $A = m\mu * \left(\frac{mK^\pm - m\mu}{mK^\pm + m\mu}\right)$; $D = 2 * \left(\frac{me^2}{m\pi^{\pm 2}} - \frac{me^2}{mp^2}\right) * \left(\frac{me^2}{mp^2 - me^2}\right) * \frac{c}{9\pi^2}$,

we may write:

$$m\Lambda_s^0 = m\Lambda_b^0 - 2m\Lambda_c^+ + Ae^{-D} \qquad [5.5]$$

$$m\Lambda_c^+ = \frac{1}{2} * (m\Lambda_b^0 - m\Lambda_s^0 + Ae^{-D}) \qquad [5.6]$$

$$m\Lambda_b^0 = 2m\Lambda_c^+ + m\Lambda_s^0 - Ae^{-D} \qquad [5.7]$$

The masses of the single elements of the family Λ are known with decreasing experimental precision starting from $m\Lambda_s^0$, for which, within the above limits, the formula holds:

$$m\Lambda_s^0 = \frac{2m\mu + e^{\pi} * me}{2\left(\frac{sinh(\theta_{m\pi^\pm})}{\theta_{m\pi^\pm}} - \frac{9}{10}\right)} \qquad [5.8]$$

[where: $\theta_{m\pi^\pm} = ln(e^{\theta_{m\pi^\pm}}) = ln(cosh(\theta_{m\pi^\pm}) + sinh(\theta_{m\pi^\pm}))$]

which is analogous to that of the neutron and the proton.

By introducing the fine structure constant α, for $m\Lambda_c^+$ we can also write:

$$m\Lambda_c^+ = \frac{2\theta_{m\pi^\pm}}{\alpha * e^{\theta_{m\pi^\pm}}} * \left(\frac{me * 2\left(cosh(\theta_{m\pi^\pm}) - \frac{9}{10}\right) * (2m\mu + e^{\pi} * me)}{m\Lambda_s^0 * 2\left(cosh(\theta_{m\pi^\pm}) - \frac{9}{10}\right) - (2m\mu + e^{\pi} * me)}\right) \qquad [5.9]$$

where $m\Lambda_s^0$ in denominator represents the part on the right in the relation [5.8].

At this point, for [5.7], we can easily write the $m\Lambda_b^0$ in a formally independent manner from the others Λ, using [5.8] and [5.9]. In the extended form, it is then:

$$\Lambda_b^0 = 2 * \frac{2\theta m\pi^\pm}{\alpha * e^{\theta m\pi^\pm}} * \left(\frac{me * 2\left(cosh(\theta m\pi^\pm) - \frac{9}{10}\right) * (2m\mu + e^{\pi} * me)}{\frac{2m\mu + e^{\pi} * me}{2\left(\frac{sinh(\theta m\pi^\pm)}{\theta m\pi^\pm} - \frac{9}{10}\right)} * 2\left(cosh(\theta m\pi^\pm) - \frac{9}{10}\right) - (2m\mu + e^{\pi} * me)}\right) +$$

$$+ \frac{2m\mu + e^{\pi} * me}{2\left(\frac{sinh(\theta m\pi^\pm)}{\theta m\pi^\pm} - \frac{9}{10}\right)} - m\mu * \left(\frac{mK^\pm - m\mu}{mK^\pm + m\mu}\right) * e^{-2*\left(\frac{me^2}{m\pi^{\pm 2}} - \frac{me^2}{mp^2}\right)*\left(\frac{me^2}{mp^2 - me^2}\right)*\frac{c}{9\pi^2}}$$

As seen in these and previous examples, the same "blocks" or groups of masses and coefficients occur in most formulas. Furthermore, given the mutual relationships between the masses, it is possible to connect each of them with each other.

PART VI – Other numerical relationships

PART VI.1 – A ratio between the masses of some particles

Numerical analyzes show that, with an excellent approximation (up to the 7th decimal digit), the following relation holds:

$$\frac{m\pi^0 * (mn - mp) * mn * mK^{\pm}}{me * m\mu * m\pi^{\pm} * \left(mn^2 + mK^{\pm 2}\right)^{\frac{1}{2}}} \cong 4 * \ln(4\pi) \qquad [6.1.1]$$

but assuming for me the value 9,109382571*10^{-31}, which in any case falls within the error margins of the data provided by NIST, the equality is valid up to the 9th decimal digit.

PART VI.2 - Modification of Koide's formula

In the so-called "Koide's formula" (Yoshio Koide, 1981) which correlates the three leptons:

$$\frac{me + m\mu + m\tau}{\left(\sqrt{me} + \sqrt{m\mu} + \sqrt{m\tau}\right)^2} \cong \frac{2}{3}$$

(= 0,666658270 with the values available), you can further approach $\frac{2}{3}$ by introducing the factor $\left(1 + e^{\theta m\pi^{\pm}} - e^{\frac{\alpha}{2}}\right)$:

$$\frac{me + m\mu + m\tau}{\left(\sqrt{me} + \sqrt{m\mu} + \sqrt{m\tau}\right)^2} * \left(1 + e^{\theta m\pi^{\pm}} - e^{\frac{\alpha}{2}}\right) = \frac{2}{3} \qquad [6.2.1]$$

(= 0,666666666 with the values available), where:

$e^{\theta_{m\pi^\pm}} = cosh(\theta_{m\pi^\pm}) + sinh(\theta_{m\pi^\pm})$, and α is the fine structure constant.

($\frac{1}{\alpha}$ = 137,035999834, while we note that $\frac{e^{\theta_{m\pi^\pm}}}{2\theta_{m\pi^\pm}}$ = 137,067247645).

PART VI.3 - Relationship between $cos(\varphi), cosh(\theta_{m\pi^\pm})$ and c

The equation:

$$cos(\varphi) = \left(\frac{2\alpha}{\mu_0} * \frac{4}{\pi} * \frac{(1 + 2\theta_{m\pi^\pm} - \alpha)^2 * e^{2\pi}}{c}\right)^{\frac{1}{2}} - cosh(\theta_{m\pi^\pm}) + 2 * \frac{9}{10} \qquad [6.3.1]$$

[where: $\theta_{m\pi^\pm} = ln(e^{\theta_{m\pi^\pm}}) = ln(cosh(\theta_{m\pi^\pm}) + sinh(\theta_{m\pi^\pm}))$]

establishes a further relationship between $cos(\varphi)$ and $cosh(\theta_{m\pi^\pm})$, involving c, α and the magnetic permeability constant μ_0. The numerical value of c can be written as:

$$c = \frac{2\alpha}{\mu_0} * \frac{4}{\pi} * \frac{(1 + 2\theta_{m\pi^\pm} - \alpha)^2 * e^{2\pi}}{(cosh(\theta_{m\pi^\pm}) + \cos(\varphi) - 2 * \frac{9}{10})^2} \qquad [6.3.2]$$

a formula which is equivalent to:

$$c = \frac{2\alpha}{\mu_0} * \frac{4}{\pi} * \frac{(1 + 2\theta_{m\pi^\pm} - \alpha)^2 * e^{2\pi}}{\left(\frac{m\mu - e^{\pi} * me}{mp} + \frac{m\mu + e^{\pi} * me}{2mn}\right)^2} \qquad [6.3.3]$$

PART VI.4 – Relationship between $sinh(\theta_{m\pi^0})$, $sinh(\theta_{m\pi^\pm})$ and $sin(\varphi)$

Since: $sinh(\theta_{m\pi^0}) = \frac{me}{m\pi^0}$, and:

$$\frac{me}{m\pi^0} = \left(\frac{5\sqrt{5}m\pi^0 - 4sin(\varphi) * mK^0}{4\sqrt{5}m\pi^0 - 2sin(\varphi) * mK^0}\right) * sinh(\theta_{m\pi^\pm}) \qquad [\text{see } 2.22]$$

then:

$$sinh(\theta_{m\pi^0}) = \left(\frac{5\sqrt{5}m\pi^0 - 4sin(\varphi) * mK^0}{4\sqrt{5}m\pi^0 - 2sin(\varphi) * mK^0}\right) * sinh(\theta_{m\pi^\pm}) \qquad [5.4.1]$$

which correlates the two hyperbolic sinuses with the trigonometric sinus.

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